

# **Report on the generation and analysis of DEMs for spatial modelling**

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## 1. Introduction

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Digital Elevation Models (DEMs) are now commonly used in many geographically oriented applications. Their production is becoming quicker and easier due to better hardware and friendlier software environments. With advances in technology it has become easy to ignore or forget a number of basic truths concerning DEMs and their characteristics. A number of these basic issues will be discussed in this report, including DEM generation, slope calculators, aspect calculators and some associated topics.

This report is an evaluation of common methods for the generation and analysis of DEMs. The discussion focuses on methods which are either already implemented or are perceived as being of reasonable complexity as to implement them manually.

### 1.1 DEM Data Formats

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There are two forms of DEM: Grid and Triangular Irregular Network (TIN). It is beyond the scope of this report to argue the case between these two data forms. Suffice to say that both these forms have various advantages and disadvantages, of which the most salient are listed below:

#### **Grid Advantages**

- Raster based and therefore familiar data structure.
- Computers can easily handle matrix manipulations.

#### **Grid Disadvantages**

- Large amount of data redundancy in areas of uniform terrain.
- Inability to adapt to areas of differing relief complexity without changing grid size.
- Can have problems deriving holes or islands.

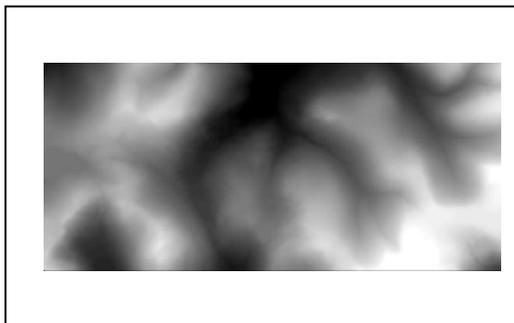


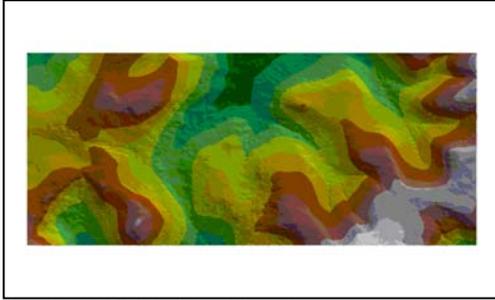
Fig 1.1 Grid DEM of the Glenshee Area

#### **TIN Advantages**

- Very little redundant data.
- Efficient data storage.
- No provision need be made for holes or islands.

#### **TIN Disadvantages**

- Can introduce a triangular discretisation hindering some forms of spatial analysis.



**Fig 1.2 TIN DEM of the Glenshee area**

Both the above figures were produced from the same data set (see appendix 1) of Glenshee

## 2. DEM Generation

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DEMs can be generated from a number of data sources (point data, orthographic imagery, contour data and grid data for example). This report will discuss methods of DEM generation from contour lines commonly available from the Ordnance Survey.

The *Profile* data set provided by the Ordnance Survey provides two levels of elevation accuracy. The majority of this data set has the contour lines at a 5m interval except in mountainous areas where the difference increases to 10m. This change over can therefore cause some contour following algorithms some confusion.

There are many ways of generating a DEM from contour lines. However most of these are heavily influenced by the discrete nature of contour information and can therefore have a stepped appearance.

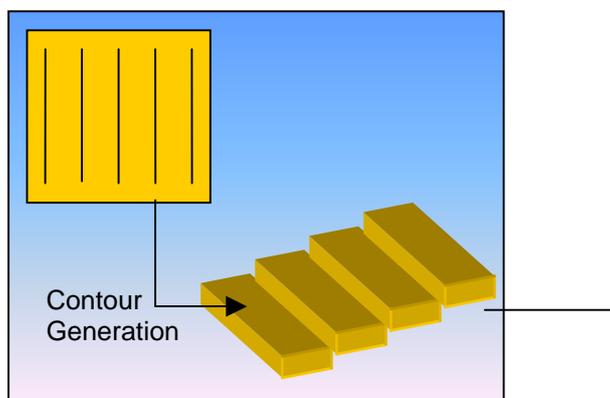


Fig 2.1 Stepped Contours – ghost-lines

This characteristic can have serious effects on viewshed and hydrographic analysis as well as being visually unappealing and misleading. These lines have been referred to as “Ghost lines” (Guth 1999) and have been detected in USGS DEMs and Ordnance Survey derived DEMs.

Other problems apparent in various packages include: algorithms not coping with local minima, tops of hills being flattened and uncharacteristic absolutely flat areas appearing on plains or valley bottoms. While it could be argued that it is unfair to expect a DEM generation algorithm to infer a landform for which it has no data, it is reasonable to expect some pattern continuation as seen with splines. These problems are also due to the quality and resolution of the input data set.

The commonly available methods of DEM generation include:

- ERDAS Imagine’s surfacing tool
- ESRI ARC/INFO’s *TOPOGRID* tool
- ESRI ARC/INFO’s *CREATETIN* then *TINLATTICE* tools
- ESRI Arcview 3.2 *Feature to TIN* then *Convert to GRID* tools

All these tools are available in house.

### 2.1 ERDAS Imagine (ERDAS 2002)

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The surfacing tool provided by ERDAS has a very simple user interface. The tool supports point, line, annotation and even raster layers as input data sources (ERDAS, 2002). The source used must have X, Y and Z components. If no Z component is available at some points then the tool will interpolate its own values.

This tool uses a TIN interpolation method, this means that where there is a Z value then this is fixed and elevations are calculated around it. This is also known as an exact method. The choice provided by the tool allows for either a linear or non-linear method. The linear method calculates a first order polynomial which defines the TIN model as angular planes. The non-linear model uses a fifth order polynomial, which results in a smooth surface. The linear method is quick, but can leave artefacts behind, whereas the non-linear model is slower but produces a smoother surface. The speed at which these algorithms derive surfaces is generally hardware dependent but have been compared in relative terms.

## 2.2 ARCINFO - TOPOGRID

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This is one of the most popular DEM surfacing tools available presently. It was developed from M.F. Hutchinson's ANUDEM (1988, 1989) and uses an iterative finite difference interpolation technique. It is optimised to calculate the best local interpolation without losing the surface continuity of global solutions. Topogrid generates a hydrologically correct elevation grid from point, line or polygon data. As water is the primary erosive force determining the shape of most landscapes, stream data can be added as an extra parameter to the interpolation (ESRI-A, 2002).

Topogrid is relatively simple to run and acts as a stand-alone, menu-driven tool inside ARCINFO. It can take a few minutes to produce a DEM but again this is generally hardware dependent. It is possible to create DEMs from contour data alone, but Profile point data can be added as a separate coverage and the addition of stream data can strengthen the surfacing process even more.

The output from this method has tended to be the most visually pleasing, avoiding the most obvious gridding patterns that can occur from contour to grid conversions. It also has the most accessible scientific literature to refer to (Hutchinson 1988, Hutchinson 1989). However it does also have some published (on the internet) criticism (Greenberg, 2002). The central implications being of a lack of accuracy in certain situations. A number of recommendations are made to minimise any risk:

- When you do not need drainage enforcement, turn it off
- Be sure to reset the tolerance to half the contour interval
- Don't expect Topogrid to yield accurate slopes in all cases
- The greatest irregularities occur when the cell size is near the contour spacing interval

## 2.3 ARCINFO - CREATETIN process

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This is a combination of two processes. Initially the contour data is converted into a TIN model and then the TIN is draped with a lattice, thereby turning it into a grid file.

The initial conversion from the contour lines to the TIN model can be conducted using either a linear or quintic interpolation technique. The linear method is similar to ERDAS's linear surfacing tool (above) in that it produces a triangular faceted surface. The quintic option also works in a similar way to ERDAS's non-linear surfacing tool, using a fifth-degree polynomial (hence quintic). However this method also allows the addition of hard break-lines. These can, for example, be used to define the edges of a lake, thereby preventing the fifth order polynomial from distorting the lake surface (ESRI-A, 2002). Again the quintic model produces a more representative result. As the density of data points increases, the two interpolation techniques begin to generate similar results.

The TINLATTICE command converts the tin model into a grid structure. Three interpolation techniques can be used for this process: nearest neighbour, bilinear interpolation and cubic convolution.

Nearest neighbour is not normally used for continuous data as it will assign the grid cell the value of the nearest TIN node. This is a better method for categorical data as it will not change the value of the input cells.

Bilinear interpolation calculates an output grid from the values of the four nearest points, based on the weighted-distance to these points. As a result, this method is suited to grids.

Cubic convolution uses the values of the sixteen nearest data points. Splines are fitted through each column of four points and another spline is fitted vertically to fit the four horizontal curves, through the data point location. This method has a tendency to smooth results. There is also an issue with what happens at the edge of the data with this method which will not be so much of a concern with the bilinear interpolation.

## 2.4 ArcView - Feature to TIN process

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This is a similar process to that described above in that the first step is to convert the contour line data into a TIN model and then convert the TIN model into a grid. In ArcView these processes can be done very simply from a menu driven environment (ESRI-B, 2002). It can also, however, be automated using either an Avenue script (ArcView 3.X) or a Visual Basic script (ArcView 8.1). This automation allows a greater level of specification to be reached as more parameters can be altered. It is worth noting that to perform these processes in ArcView it is necessary to have both the Spatial Analyst and 3D Analyst extensions available.

## 2.5 DEM Generation Comparison

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Six DEMs were generated from the same data set. The methods used to generate these surfaces were:

1. Linear ERDAS surfacing tool
2. Non Linear ERDAS surfacing tool
3. ArcInfo Topogrid tool
4. ArcInfo Linear CREATETIN process
5. ArcInfo Quintic CREATETIN process
6. ArcView Feature to TIN process

A number of ways have been suggested by the literature for comparing DEMs. These are generally visual assessments of different forms of the data model.

The two visual comparisons to be made here are using a hillshading algorithm and using the Fourier transform.

### **Hillshading**

This method has been implied by Guth, 1999, and suggested by Wise 2001. They propose that by using one of a number of similar hillshading algorithms, periodic patterns can be distinguished in the generated surfaces, implying the presence of Guth's Ghost lines or other artefacts left over from the interpolation process.

The algorithm used for this comparison was that featured in ERDAS Imagine's relief shading tool. The details of the exact algorithm are irrelevant since the algorithm is calculating the amount of shadow in a given area from a given sun direction. This allows unnatural patterns to be picked up more easily. For this calculation the following factors were used:

- Solar Azimuth = 315
- Solar Elevation = 45
- Ambient Light = 0.00

The literature has suggested that the first two quantities are the preferred angles for human perception (ESRI-C, 2001). The lack ambient light means that the highest contrast image will be produced.

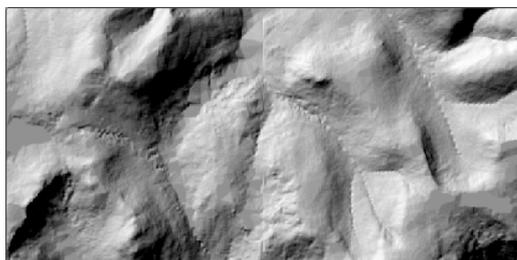


Fig 2.2 Hill shaded linear ERDAS surfacing tool

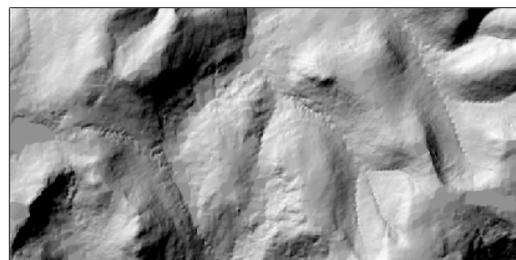


Fig 2.5 Hill Shaded ArcInfo linear CREATETIN process

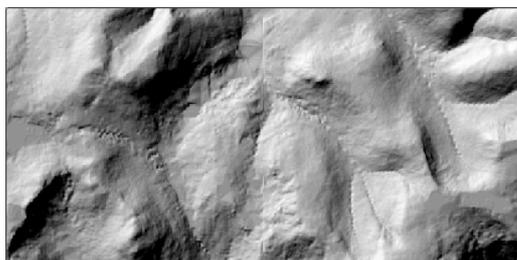


Fig 2.3 Hill shaded non-linear ERDAS surfacing tool

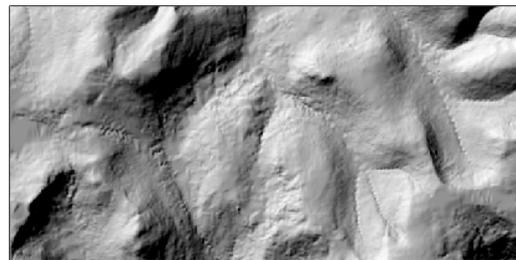


Fig 2.6 Hill Shaded ArcInfo Quintic CREATETIN process

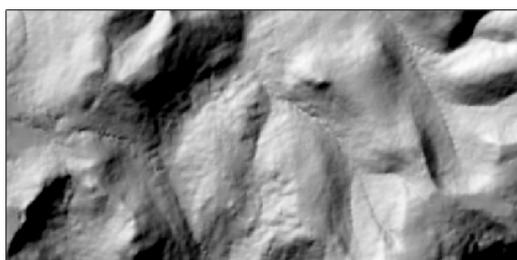


Fig 2.4 Hill shaded ArcInfo Topogrid tool



Fig 2.7 Hill shaded ArcView feature to TIN process

The main differences are between the linear and non-linear surfaces, principally seen in the level of detail. However the surface differences also indicate stepping artefacts up riverbeds. Interestingly the join between the two coverages (see appendix 1) can be detected in some of the images, most markedly in the two ERDAS DEMs. Some artefacts from the triangulation process can also be easily observed. The surface least affected by the artefacts is the Topogrid tool output. An example closer comparison is included in appendix 2.

### Fourier Analysis

This is a more abstract method of image processing and analysis. Fourier analysis works on the imagery in the frequency domain, separating an image into its various spatial frequency components (Jensen, 1996). This allows periodic data to be detected more easily. It is commonly used to destripe Landsat data and has been used to investigate DEMs (Guth 1999).

In this case the DEMs have been transformed into a form ready to be edited in ERDAS's "Fourier editor" and then optimised using the Fourier magnitude calculation to enable the Fourier image to be viewed in a normal viewer. This could not happen previously as each 'pixel' in a Fourier image is made up of a complex number, which has both a real and an

imaginary component, causing some issues for the display device. Imagine combines these in a root-sum of squares operation for viewing. The process by which Fourier analysis works is to separate an image into its component frequencies. These are then displayed in a two dimensional space with the lower frequencies plotted in the centre, while the progressively higher frequencies are plotted further out. Features that trended horizontally in the input data would be characterised vertically on the Fourier plot and vice versa.

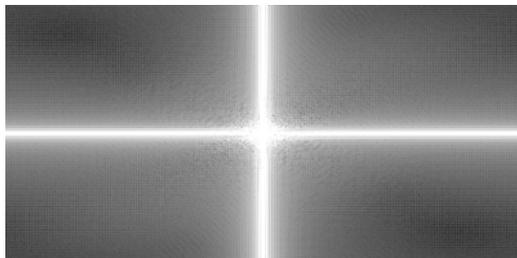


Fig 2.8 Fourier image of Linear ERDAS surfacing tool

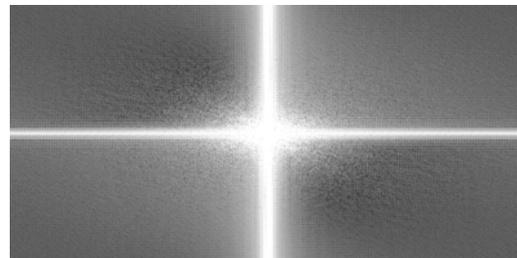


Fig 2.11 Fourier image of ArcInfo linear CREATE TIN process

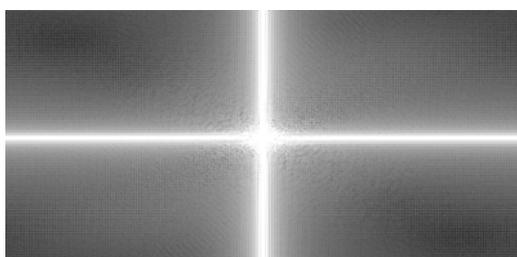


Fig 2.9 Fourier image of non-linear ERDAS surfacing tool

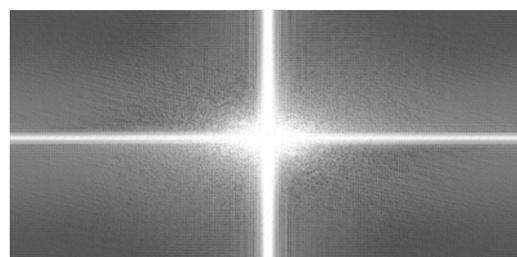


Fig 2.12 Fourier image of ArcInfo quintic CREATE TIN process

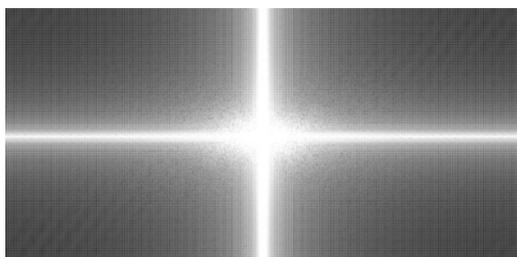


Fig 2.10 Fourier image of ArcInfo Topogrid tool

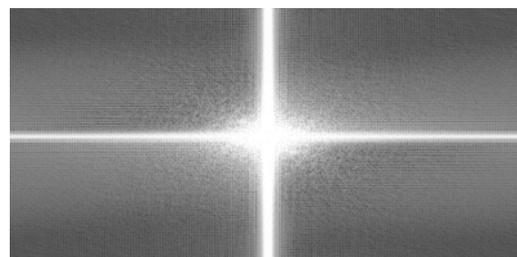


Fig 2.13 Fourier image of ArcView feature to TIN process

These images show that there are significant amounts of periodic artefact present in all the surfaces. This will be due in part to the gridding process to which all the surfaces will have been subjected. However there are some interesting differences between the non-linear ERDAS surface and the ArcInfo quintic surface. The Fourier images are significantly different despite the processes being described as identical. The same is true for the ERDAS linear and ArcInfo Linear images. This is likely to be due to the differing computational tolerances to which the ERDAS and ESRI software adhere. The Topogrid tool shows the greatest amount of lower frequency activity. This implies that there are fewer small details present in this DEM. The further implication of this is that as the input data is not of a very high resolution and the original is not highly detailed then the Topogrid output is therefore producing fewer artefacts and a smoother surface.

## 2.6 Recommendation

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For the purposes of fast, easily repeatable results ARCIINFO's Topogrid tool is recommended. This process has proved the most visually pleasing while also being more scientifically reliable. However all methods outlined here will provide reasonable results if used wisely. It is worth noting a few rules of thumb for efficient generation of DEMs from contour lines:

- This process is highly processor intensive, so each DEM generated could take some minutes to produce.
- The profile data is a good resource for building DEMs if merging a group of tiles together ensure they have the same contour interval, if they do not then aggregate the 5m set or interpolate extra lines in the 10m set.
- Use as many data sources as possible to produce the DEM i.e. if using Topogrid then use the spot heights as well as the contour data and use any available stream data to strengthen the interpolation.
- Build DEMs to the necessary resolution otherwise it can be easy to disable oneself. Being excessive with scale (i.e. using a 1m grid where a 20m grid is enough) will mean that the DEM generation process will take significantly longer and subsequent operations will be slowed down noticeably.
- When using alternative data sources for DEM generation, be aware of its meta-data and the standards to which the elevation data has been collected.

### 3. Slope Calculators

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The slope of a DEM is generally identified as the maximum rate of change in value from each cell to its neighbours (the fall line). The output grid from this process should be measured as angle of slope or percent slope. This means that a 45° would be a slope of 100% and a 90° slope would be infinity. In some software this has been simplified to 90° equalling 200%. Slope is a commodity that is generally more accurately measured from Grid based DEMs, although this is resolution dependent.

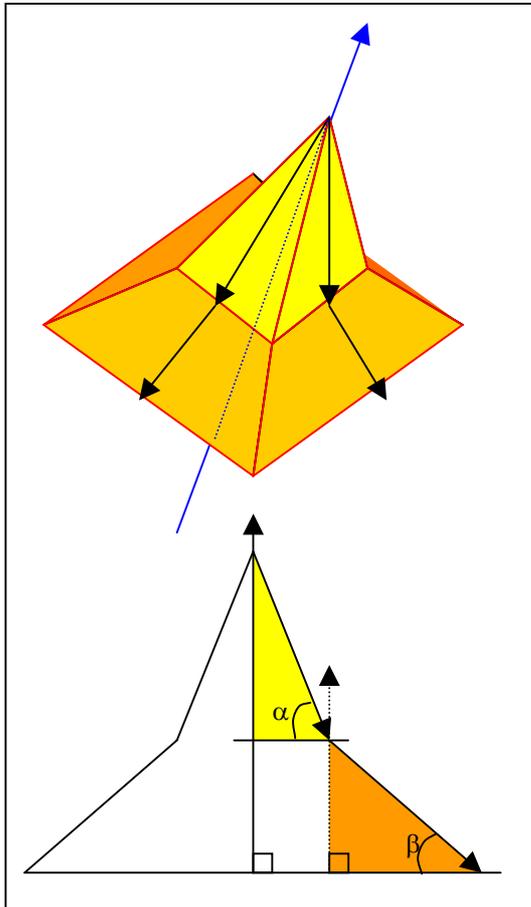


Fig 3.1 Slope angles, TIN example

#### 3.1 Grid Slope Algorithms

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Jones (1998) describes eight algorithms for calculating slope of a DEM. Most of these methods differ only in the number of grid cells used and the weightings applied to each value. However only a few of these have been implemented into commercial software and these have tended to be the algorithms favoured by the literature. Jones found that a second-order, finite-difference algorithm (Fleming and Hoffer 1979, Zevenbergen and Thorne 1987) to be the 'best'. However, Horn's (1981) third-order, finite-difference method, which has been implemented in ARC/INFO came out as second best. Skidmore (1989) has also provided an insight into slope and aspect calculators, although his findings do not completely agree with those of Jones there does seem to be a consensus that Horn's method is one of the most reliable methods. Skidmore even suggests that the third order finite difference method is optimal for calculating gradient from a gridded DEM. Burrough and McDonnell (2000) review the subject and suggest that both algorithms have their relative merits as they both use different numbers of data points to reach their answers.

For completeness, three algorithms are described here. The third, ERDAS Imagine's method, has not been discussed above, as there is no literature associated with it and was apparently written internally.

### 3.2 Zevenbergen and Thorne's Method - Slope

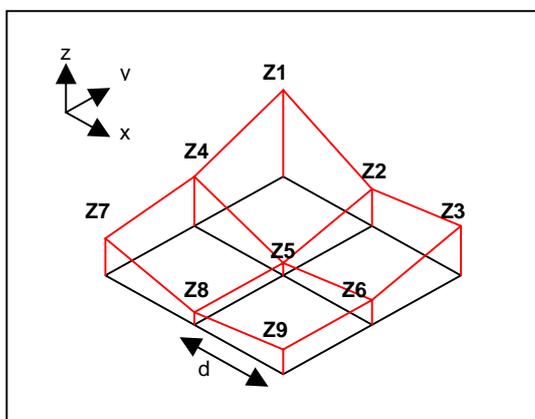


Fig 3.2 Zevenbergen and Thorne's model

The slope surface is derived from the quadratic equation:

$$Z = Ax^2y^2 + Cxy^2 + Dx^2 + Ey^2 + Fxy + Gx + Hy + I$$

Eqn 3.1

The nine parameters are determined from the nine elevations (Z-values above) from the 3x3 window using Lagrange polynomials:

$$A = [(Z_1 + Z_3 + Z_7 + Z_9) / 4 - (Z_2 + Z_4 + Z_6 + Z_8) / 2 + Z_5] / L^4$$

Eqn. 3.2

$$B = [(Z_1 + Z_3 - Z_7 - Z_9) / 4 - (Z_2 - Z_8) / 2] / L^3$$

Eqn. 3.3

$$C = [(-Z_1 + Z_3 - Z_7 + Z_9) / 4 - (Z_4 - Z_6) / 2] / L^3$$

Eqn. 3.4

$$D = [(Z_4 + Z_6) / 2 - Z_5] / L^2$$

Eqn. 3.5

$$E = [(Z_2 + Z_8) / 2 - Z_5] / L^2$$

Eqn. 3.6

$$F = (-Z_1 + Z_3 + Z_7 - Z_9) / 4L^2$$

Eqn. 3.7

$$G = (-Z_4 + Z_6) / 2L$$

Eqn. 3.8

$$H = (Z_2 + Z_8) / 2L$$

Eqn. 3.9

$$I = Z_5$$

Eqn. 3.10

The slope index is found by differentiating equation 3.1 and solving the resultant equation for the centre point. Although conceptually this method uses a three by three window to calculate the slope, the mathematics simplifies to only using four data points.

$$\text{SLOPE} = \partial Z / \partial S = G \cos \theta + H \sin \theta$$

Eqn. 3.11

$$\therefore \text{SLOPE} = - (G^2 + H^2)^{1/2}$$

Eqn. 3.12

The negative sign indicates that the slope is a down slope and, conventionally is ignored. Implementation of all methods is described later.

### 3.3 Horn's Method - Slope

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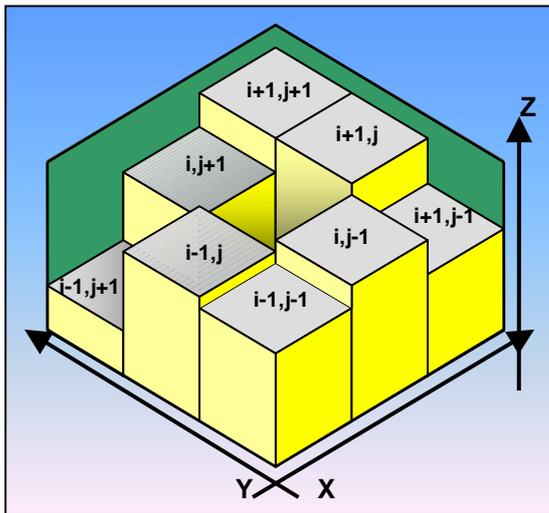


Fig 3.2 Horn's Model

East-West slope:

$$[\delta Z / \delta X]_{i,j} = [(Z_{i+1,j+1} + 2Z_{i+1,j} + Z_{i+1,j-1}) - (Z_{i-1,j+1} + 2Z_{i-1,j} + Z_{i-1,j-1})] / 8\delta X$$

Eqn. 3.13

North-South slope:

$$[\delta Z / \delta Y]_{i,j} = [(Z_{i+1,j+1} + 2Z_{i,j+1} + Z_{i-1,j+1}) - (Z_{i+1,j-1} + 2Z_{i,j-1} + Z_{i-1,j-1})] / 8\delta Y$$

Eqn. 3.14

$$\text{SLOPE} = [(\partial z / \partial x)^2 + (\partial z / \partial y)^2]^{1/2}$$

Eqn. 3.15

As mentioned earlier Horn's method is ready implemented in ARC/INFO, it is also the method used by ArcView to calculate slopes in the Spatial Analyst extension.

### 3.4 ERDAS Imagine's Method - Slope

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Imagine views the relationship between the percentage and degrees of slope slightly differently from the other algorithms:

- 45° is considered a 100% slope
- 90° is considered a 200% slope
- slopes less than 45° fall within 1 – 100%
- slopes between 45° and 90° are expressed as 100 – 200%

<b>A</b>	<b>B</b>	<b>C</b>
<b>D</b>	<b>E</b>	<b>F</b>
<b>G</b>	<b>H</b>	<b>I</b>

$\Delta X_1 = C - A$	$\Delta Y_1 = A - G$
$\Delta X_2 = F - D$	$\Delta Y_2 = B - H$
$\Delta X_3 = I - G$	$\Delta Y_3 = C - A$
$\Delta X = (\Delta X_1 + \Delta X_2 + \Delta X_3) / 3 \times X_s$	
$\Delta Y = (\Delta Y_1 + \Delta Y_2 + \Delta Y_3) / 3 \times Y_s$	

A – I are the elevation values of the pixels in the 3x3 window  
 $X_s$  is the pixel size in the X direction  
 $Y_s$  is the pixel size in the Y direction

The slope at pixel X,Y is calculated as:

$$S = \sqrt{(\Delta X)^2 + (\Delta Y)^2} / 2$$

If  $S \leq 1$  percent slope =  $S \times 100$

If  $S > 1$  percent slope =  $200 - 100/S$

Slope in degrees =  $\tan^{-1}(S) \times 180/\pi$

### 3.5 Comparison and Conclusions

In both Zevenbergen and Thorne's method and Horn's the final result still needs to be manipulated into a commonly recognisable quantity. It can either be turned into a percentage or an angle. This is done in a similar way to the ERDAS Imagine method except there is no differentiation between the slope being above or below 1.

Percent slope = SLOPE  $\times$  100

Eqn. 3.16

Angle of slope =  $\tan^{-1}$  (SLOPE)

Eqn. 3.17

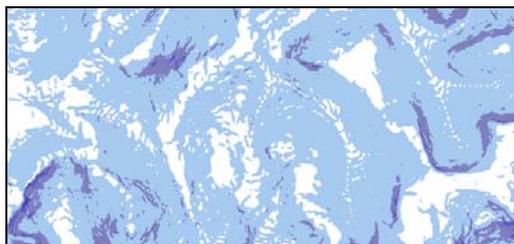


Fig 3.3 Slope surface of Glenshee area

Where the darker areas show regions of greatest slope and the lighter areas regions of least slope

It is reassuring to know that the two best algorithms (Jones, 1998) are already used in various commercial GIS applications. The general rule of thumb concerning which algorithm to use is:

- Zevenbergen and Thorne's method is better for smoother surfaces
- Horn's method is better for rough surfaces.

In ESRI products Horn's method tends to be the default and perhaps seemingly only option as regards the DEM analysis functions, however there are scripts and extensions available from their web-site (<http://arcscripts.esri.com/>) which allow a choice to be made between these two preferred methods.

Imagine's method is included here as it is readily available, but there is no literature to support it other than ERDAS's own help files.

### 3.6 TIN Slope Calculators

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It is less of a problem to calculate slope from a TIN as it can easily be calculated from the angle of the hypotenuse of each triangle from the horizontal (the 'adjacent'). Each face will have a constant slope. This can be done easily through ArcView and ARCINFO. However a slope calculated from TINs is likely to be less representative due to the triangular faces being of constant slope.

## 4. Aspect Calculators

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Aspect is the down-slope direction of the maximum rate of change in value from each cell to its neighbours. This is adapted to be measured in degrees according to directions on a compass.

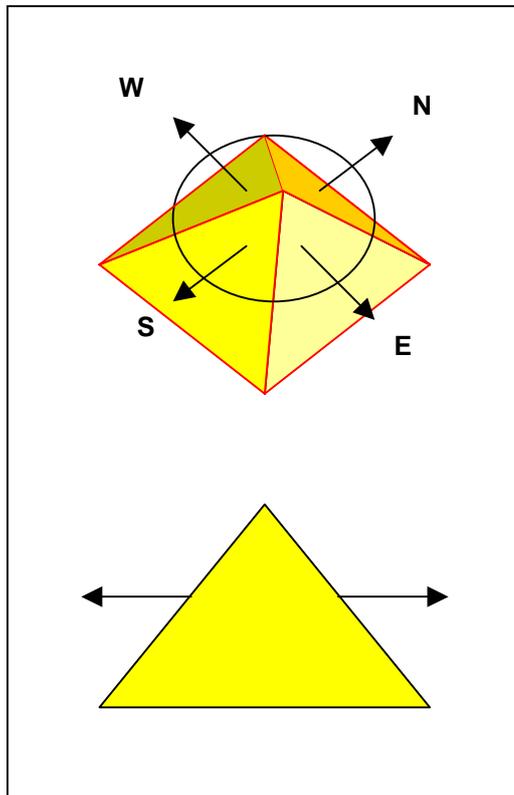


Fig 4.1 Aspect angles, TIN example

The aspect is relatively easily calculated once the slope has been determined. Therefore the same discussions as above are relevant. It is logical for reasons of continuity to use the complimentary method for aspect as was used for slope and vice versa.

### 4.1 Zevenbergen and Thorne's Method - Aspect

---

The aspect is found by differentiating equation 3.11 to find its minimum.

$$\partial \text{SLOPE} / \partial \theta = -G \sin \theta + H \cos \theta = 0$$

Eqn. 4.1

$$\theta = \arctan (-H / -G)$$

Eqn. 4.2

### 4.2 Horn's Method - Aspect

---

The final steps in finding the aspect of a slope using Horn's method is very similar to the final step's in that of Zevenbergen and Thorne's. The gradient is calculated from:

$$\text{Tan (ASPECT)} = (\partial z / \partial x) / (\partial z / \partial y)$$

Eqn. 4.3



Fig 4.2 Aspect surface of Glenshee area

Both these methods are subject to the “arctan problem” (see appendix 3). This simply means that care must be taken when calculating the aspect angle. It is important be aware in which quadrant of the “compass” the correct aspect angle lies.

### 4.3 ERDAS Imagine’s Method

---

Again this method is similar to those used above. Aspect is expressed in degrees, clockwise from 0 to 360, where due north is 0. A value of 361 denotes a flat surface.

$$\text{ASPECT} = \theta + 180$$

$$\text{where, } \theta = \tan^{-1}(\Delta X / \Delta Y)$$

The 180 are added on here to compensate for the arctan problem as described above and in appendix 3.

## 5. Implementation

---

This section describes a basis for the implementation of the slope and aspect algorithms described above, inside a software environment. This is aimed at those who want to code these DEM analysis tools inside software which does not already have the algorithms present (for example Smallworld GIS package). It may also help those who wish to write their own scripts for DEM analysis inside environments which have other algorithms implemented.

For the purposes of this description it is assumed that the DEMs being used are in a raster (grid) format. Problems associated with file formats will be discussed later.

### 5.1 Zevenbergen and Thorne's Method

---

This is a second order, finite difference algorithm, which computes slope and aspect from the nearest four elevation points on a grid. The coefficients of the partial quadratic equation are deduced from a trend surface, which passes exactly through the nine elevation points. The nine elevation points refer to a three by three window, which is passed over DEM to derive the slope surface.

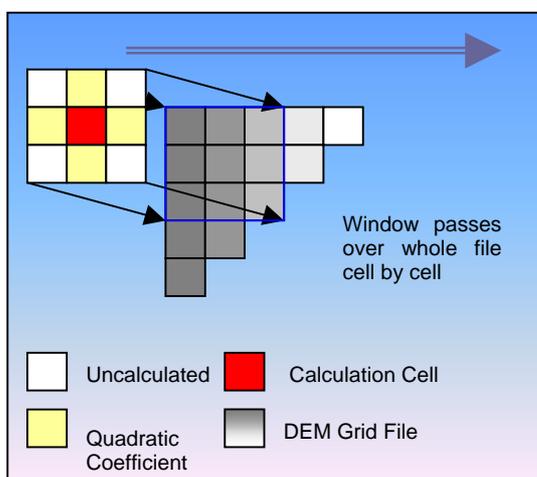


Fig 5.1 3x3 window using Zevenbergen and Thorne's method

As discussed in section 3.2 Zevenbergen and Thorne's method calculates the slope from only four data points. These points amount to the North-South and East-West Points. From these four points two quantities are derived, describing the angular trend in the North-South (H) and East-West (G) directions:

$$G = (-Z_4 + Z_6) / 2L \quad (\text{East-West})$$

Eqn. 3.8

$$H = (Z_2 - Z_8) / 2L \quad (\text{North-South})$$

Eqn. 3.9

The slope is then calculated using:

$$\text{SLOPE} = - (G^2 + H^2)^{1/2}$$

Eqn. 3.12

The aspect is also calculated from G and H:

$$\text{ASPECT} = \theta = \arctan (-H / -G)$$

The positive or negative natures of G and H have a major bearing on the angle of aspect due to the arctan problem (see appendix 2)

## 5.2 Zevenbergen and Thorne's Method - Example

---

10	20	25
22	<b>Z</b>	25
20	24	18

Fig 5.2 Example data set

The pixel for which slope is being calculated has been shaded, elevations of the pixels are given in metres. The pixel size has been set at 10m

SLOPE Calculation:

$$\begin{aligned}G \text{ (East-West)} &= (-22+25)/2*10 \\ &= 3/20 \\ &= 0.15\end{aligned}$$

$$\begin{aligned}H \text{ (North-South)} &= (20-24)/2*10 \\ &= -4/20 \\ &= -0.2\end{aligned}$$

$$\begin{aligned}\text{SLOPE} &= -(0.15^2+0.2^2)^{1/2} \\ &= -(0.0225 + 0.04)^{1/2} \\ &= -0.25\end{aligned}$$

$$\text{Angle of slope} = \tan^{-1} (\text{SLOPE})$$

$$= \tan^{-1} (0.25) = 14.03^\circ$$

ASPECT Calculation:

$$\begin{aligned}\theta &= \arctan (-H / -G) \\ &= \arctan (0.2/-0.15) \\ &= -53.1\end{aligned}$$

$$\text{ASPECT} = -53.1 + 180$$

ASPECT = 126.9° from the positive x-axis  
(see appendix 3 – the arctan problem)

## 5.3 Horn's Method

---

This is a third-order, finite-difference technique that uses unequal weighting coefficients for the nearer elevation values. These weightings are proportional to the reciprocal of the square of the distance from the kernel centre. This technique takes into account the eight closest grid points surrounding the point of analysis.

In practical terms, a three by three window is passed over the grid file. The central pixel is subject to the algorithm, with the surrounding pixels providing the coefficients for the calculation.

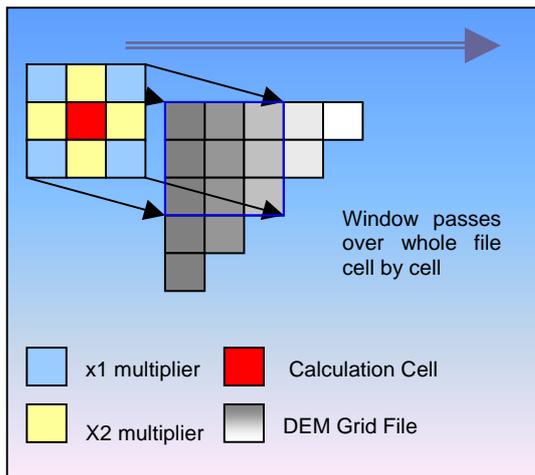


Fig 5.2 3x3 window using Horn's method

Horn's method means that the weightings of the coefficients are different depending on their spatial relationship to the central pixel. The algorithm is described as follows:

East-West gradient:

$$[\delta Z / \delta X]_{i,j} = [(Z_{i+1,j+1} + 2Z_{i+1,j} + Z_{i+1,j-1}) - (Z_{i-1,j+1} + 2Z_{i-1,j} + Z_{i-1,j-1})] / 8\delta X$$

North-South gradient:

$$[\delta Z / \delta Y]_{i,j} = [(Z_{i+1,j+1} + 2Z_{i,j+1} + Z_{i-1,j+1}) - (Z_{i+1,j-1} + 2Z_{i,j-1} + Z_{i-1,j-1})] / 8\delta Y$$

Where Z is the central pixel, i describes the number of rows, j describes the number of columns (for example, see below).

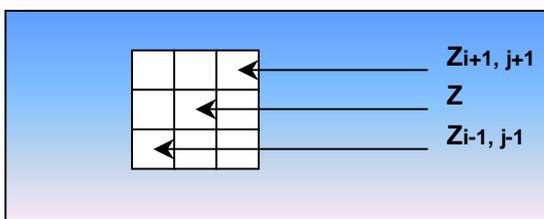


Fig 5.3 Horn's 3x3 window notation

$\delta X$  and  $\delta Y$  describe the size of the pixel in the X and Y directions for example with a Landsat image (outside the panchromatic or thermal bands) this would be 30m in both directions.

The gradient (G) is then defined as:

$$\tan G = \sqrt{[(\delta Z / \delta X)^2 + (\delta Z / \delta Y)^2]}$$

while aspect (A) is defined as:

$$\tan A = (\delta Z / \delta X) / (\delta Z / \delta Y)$$

### 5.4 Horn's Method – Example

---

10	20	25
22	<b>Z</b>	25
20	24	18

Fig 5.4 Example data set

The pixel for which slope is being calculated has been shaded, elevations of the pixels are given in metres. The pixel size has been set at 10m

East-West =

$$\frac{[(25 + 2(25) + 18) - (10 + 2(22) + 20)]}{8(10)}$$

$$= 0.175$$

North-South =

$$\frac{[(25 + 2(20) + 10) - (18 + 2(24) + 20)]}{8(10)}$$

$$= -0.1375$$

SLOPE (S) is:

$$\tan S = \sqrt{[(0.175)^2 + (-0.1375)^2]}$$

$$\tan S = 0.223$$

$$\text{SLOPE} = 12.54^\circ$$

Aspect (A) is:

$$\tan A = 0.175 / -0.1375$$

$$\tan A = -1.27$$

$$A = 51.82$$

$$\text{ASPECT} = -51.82 + 180$$

$$\text{ASPECT} = 128.18^\circ \text{ from the positive x-axis}$$

(see appendix 3 – the arctan problem)

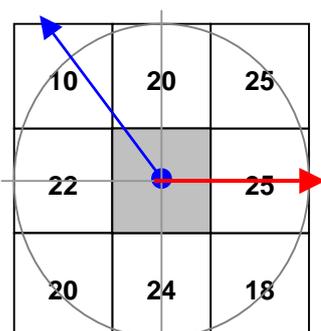


Fig 5.5 approximate angle of aspect

### 5.5 ERDAS Imagine's Method

---

Again the 3x3 window is used to calculate the DEM indices. However, ERDAS Imagine uses an unweighted differencing method. This takes the difference for each column in the X direction and the difference for each row in the Y direction and averages them into a single X and a single Y value.

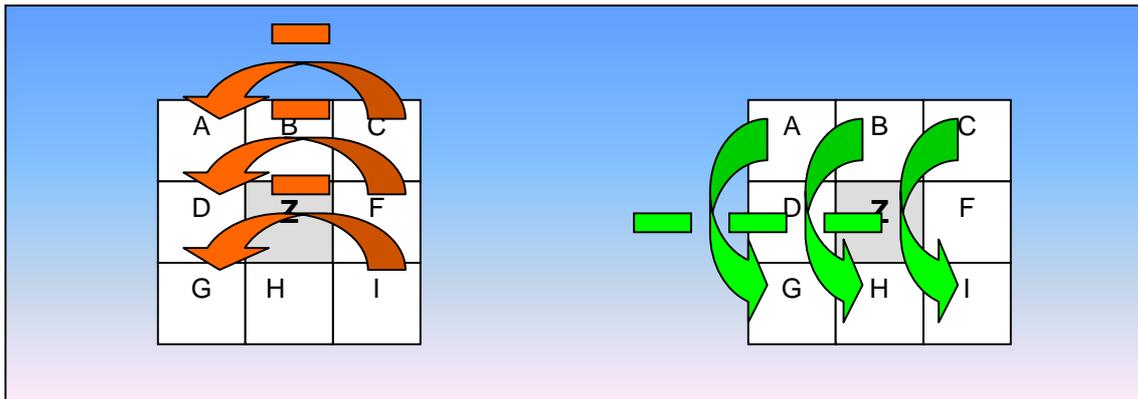


FIG 5.6 direction of ERDAS Imagine's Differencing Method

$$\begin{aligned} \Delta X_1 &= C - A & \Delta Y_1 &= A - G \\ \Delta X_2 &= F - D & \Delta Y_2 &= B - H \\ \Delta X_3 &= I - G & \Delta Y_3 &= C - I \end{aligned}$$

$$\Delta X = (\Delta X_1 + \Delta X_2 + \Delta X_3) / 3 \times X_s$$

$$\Delta Y = (\Delta Y_1 + \Delta Y_2 + \Delta Y_3) / 3 \times Y_s$$

Where:

A - I are the elevations of the pixels encompassed by the 3x3 window

$X_s$  is the pixel size in the X direction

$Y_s$  is the pixel size in the Y direction

The slope is then calculated from:

$$S = \sqrt{(\Delta X)^2 + (\Delta Y)^2} / 2$$

If  $S \leq 1$  percent slope =  $S \times 100$

If  $S > 1$  percent slope =  $200 - 100/S$

$$\text{Slope in degrees} = \tan^{-1}(S) \times 180/\pi$$

The aspect is then described by:

$$\text{ASPECT} = \theta + 180$$

$$\text{where, } \theta = \tan^{-1}(\Delta X / \Delta Y)$$

This method describes the aspect in degrees, clockwise from 0 (due North) to 360. Imagine uses the value 361 to denote a flat surface such as a water body.

## 5.6 ERDAS Imagine's Method – Example

---

10	20	25
22	<b>Z</b>	25
20	24	18

FIG 5.7 Example Data set

Assuming a square pixel size of 10m

$$\Delta X_1 = C - A = 25 - 10 = 15$$

$$\Delta Y_1 = A - G = 10 - 20 = -10$$

$$\Delta X_2 = F - D = 25 - 22 = 3$$

$$\Delta Y_2 = B - H = 20 - 24 = -4$$

$$\Delta X_3 = I - G = 18 - 20 = -2$$

$$\Delta Y_3 = C - A = 25 - 18 = 7$$

$$\Delta X = (15 + 3 + -2) / 3 \times 10 = 0.533$$

$$\Delta Y = (-10 + -4 + 7) / 3 \times 10 = -0.233$$

$$\begin{aligned} \text{SLOPE} &= \sqrt{(0.533^2 + -0.233^2)} / 2 \\ &= \sqrt{0.338} / 2 \\ &= 0.29 = 29\% \end{aligned}$$

The angle of this slope is (remembering to work in radians):

$$\tan^{-1}(0.29) \times 180/\pi = 16.17^\circ$$

The aspect of the slope is (again working in radians):

$$\theta = \tan^{-1}(0.533 / -0.233) = 1.158$$

Convert  $\theta$  to degrees = 66.387

ASPECT =  $\theta + 180 = 66.4 + 180 = 246.4^\circ$  from the north (i.e. approximately North West)

## 5.7 Conclusion

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Both Zevenbergen and Thorne's, and Horn's methods are effective in calculating the slope and aspect of a DEM. They do however produce slightly different results. It has been suggested in the literature that Zevenbergen and Thorne's method is more suited to smooth, rolling landscapes as it may have the tendency to smooth data as it uses only four data points. Horn's method is thought to be generally better for rougher terrain where there may be a real difference made by the eight input points rather than four. The computing times will be pretty similar on most modern computers. ERDAS Imagine's method suggests similar results, but has tackled the aspect problem in a slightly different way. The literature for this method does not describe any conceptual basis for this method but it does however give reasonable results.

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## Further Reading

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## Glossary

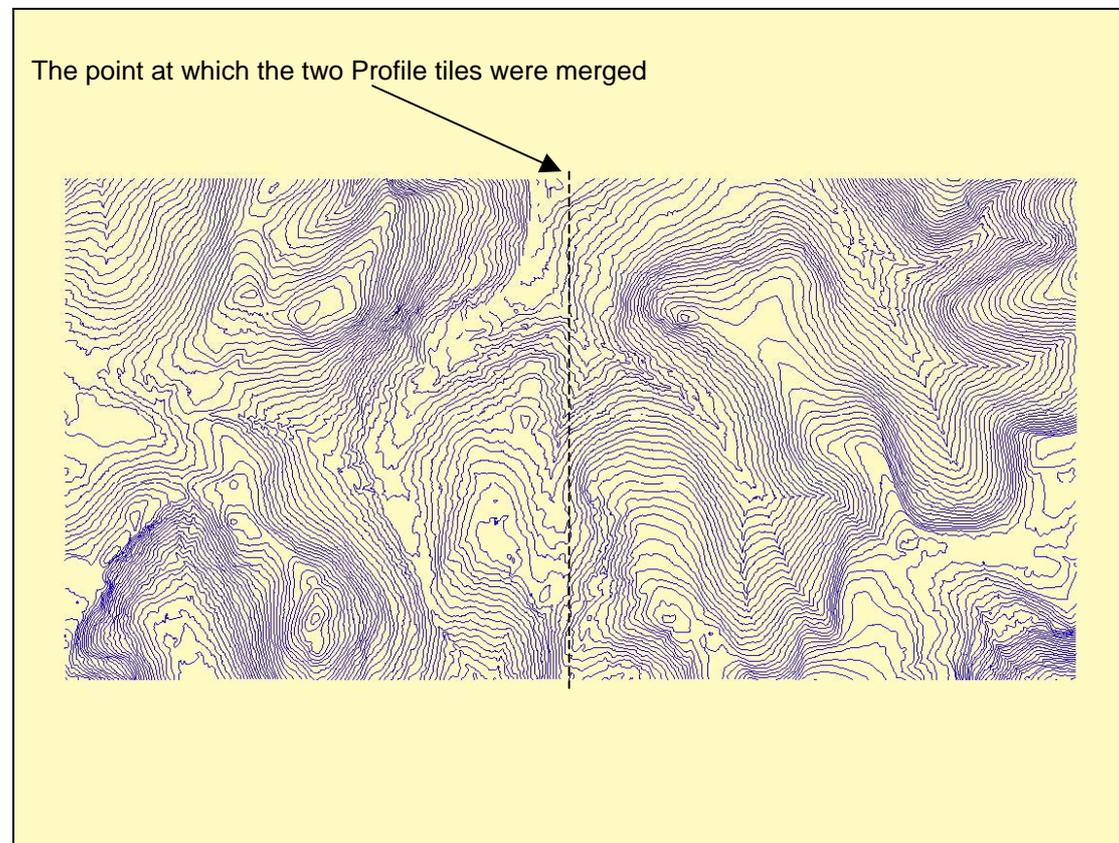
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<b>Artefact</b>	-An artificial feature left over from a process (i.e. DEM generation)
<b>Aspect</b>	-The direction (compass bearing) of the gradient
<b>Bilinear interpolation</b>	-Grid resampling technique using weighted values of the four nearest points
<b>Contour lines</b>	-Line indicating a specific height (iso-line)
<b>Cubic Convolution</b>	-Grid resampling technique using the values of sixteen surrounding points. Has a tendency to smooth results
<b>DEM</b>	-Digital Elevation Model
<b>DTM</b>	-Digital Terrain Model
<b>Fourier Analysis</b>	-A method to investigate periodic signatures in remote sensing data
<b>Frequency Domain</b>	-The domain in which Fourier analysis is conducted
<b>Ghost lines</b>	-Stepping effect that can sometimes be associated with discrete nature of contour lines
<b>Gradient</b>	-Maximum rate of change in elevation from one grid cell to another
<b>Grid</b>	-An array of pixels or points
<b>Hydrographic analysis</b>	-Analysis of movement of fluids through the landscape
<b>Lagrange Polynomials</b>	-A mathematical interpolation process
<b>Landsat</b>	-A US satellite constellation which has been functioning for over thirty years
<b>Linear</b>	-To do with a straight line, in this case a linear method of DEM generation with use a TIN model and generate flat faces to describe the surface
<b>Nearest neighbour</b>	-Grid resampling technique that assigns a value according to the nearest data point
<b>Non-linear</b>	-Not to do with a straight line, a non-linear DEM generation method will develop a model which will have curved surfaces, therefore being more representative
<b>TIN</b>	-Triangular Irregular Network
<b>Ordnance Survey</b>	-UK national mapping agency
<b>Quintic</b>	-Fifth order differential
<b>Resolution</b>	-The ratio between a pixel and its equivalent ground coverage
<b>Slope</b>	-In this report slope and gradient are used interchangeably
<b>Spot height</b>	-Point with a specific height attached
<b>USGS</b>	-United States Geological Survey
<b>Viewshed analysis</b>	-Analysis of visible areas from a point on a landscape

## Appendix 1 - Glenshee Data Set

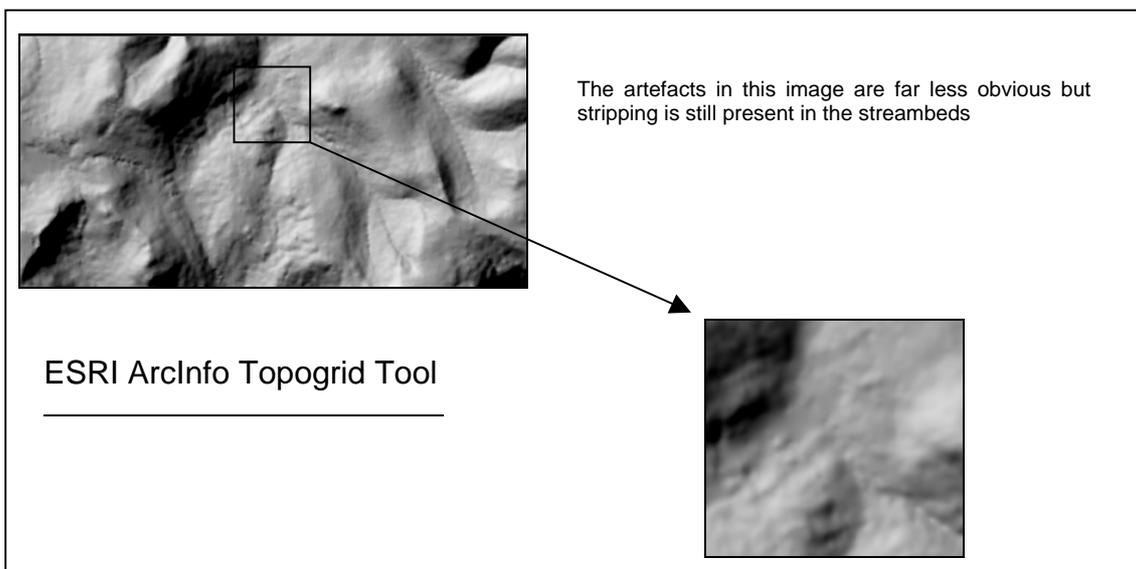
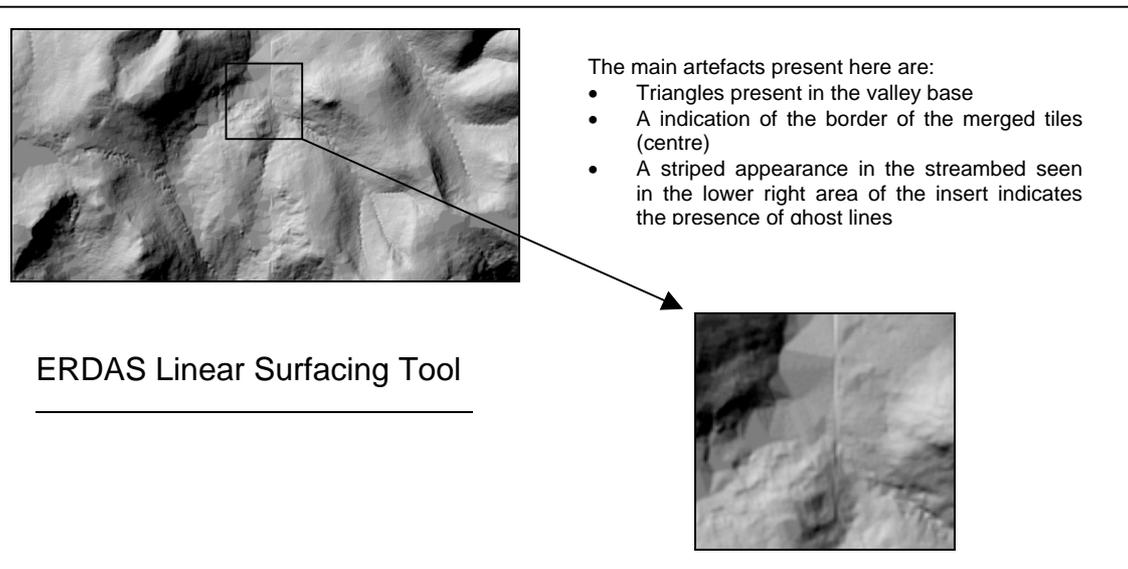
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This is the contour data source from which all surfaces in this report have been generated. This is a merged and clipped coverage of two ordnance survey profile tiles describing the Glenshee area.



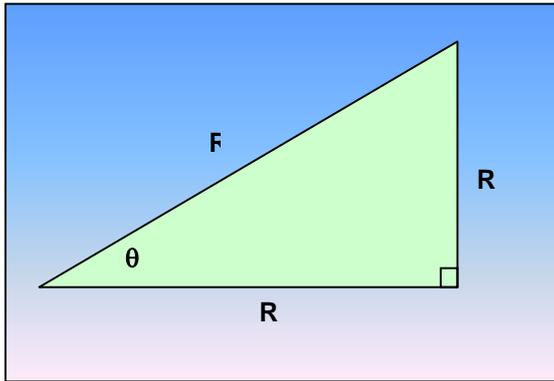
## Appendix 2 - Closer Examination of Hillshaded DEMs

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This is an example comparison, which is in some respects unfair as one method is linear and in effect the Topogrid tool is a non-linear method. However it shows the difference in quality between two DEM generation methods. In this case the artefacts present in the ERDAS linear surfacing tool can be easily seen especially in streambeds and in valley bottoms. These are generally remnants of the triangulation process, which was the first step in this tool's surfacing process. The Topogrid tool has produced a DEM that appears to be more blurred however in general this is a more representative description of Glenshee. There are still artefacts but these could be minimised with the addition of stream data and the spot heights.

## Appendix 3 - The Arctangent Problem

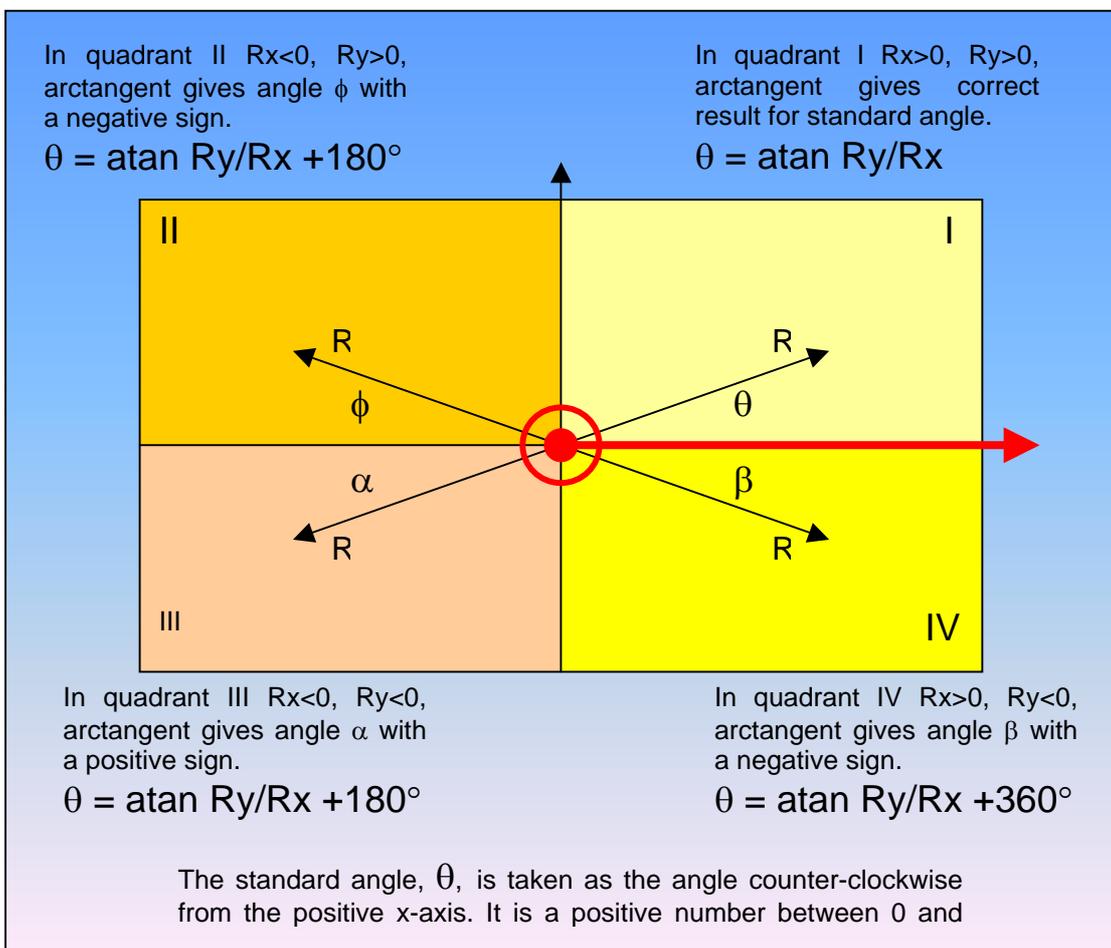


In the right-angled triangle shown, the angle  $\theta$  can be calculated from:

$$\theta = \text{atan}Ry/Rx = \text{arctan}Ry/Rx = \tan^{-1}Ry/Rx$$

(Different notations for the same function)

A common problem arises with vector addition when the components of the resultant vector are found is that the arctan function in calculators and computer languages cannot distinguish the quadrant of the angle.



Example using Zevenbergen and Thorne's method and notation:

If  $G = 0.15$  and  $H = -0.2$

Then

$$\theta = \text{arctan } -H / -G = \text{arctan } 0.2 / -0.15 = \text{arctan } -1.33 = -53.13$$

As  $0.2 / -0.15, R_x < 0$  and  $R_y > 0$ , then this solution is in quadrant II and the final answer is:  $-53.13 + 180 = 126.87^\circ$  anti-clockwise from the positive x-axis.

Approximately this angle:

